HB-003-001544

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

May / June – 2017

Statistics : S - 503

(Statistical Inference) (New Course)

Faculty Code : 003
Subject Code : 001544

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) Students can use their own scientific calculator.
(3) Students can demand log table on request.

1 Filling the blanks and short questions : **20**

(1) An estimator T_n which is most concentrated about parameter θ is the ________ estimator.

(2) Estimation is possible only in case of a ________

(3) If an estimator T_n converges in probability to the parametric function $\tau(\theta), T_n$ is said to be a ________ estimator.

(4) $\sum \frac{X_i}{n}$ *for* $i = 1, 2, 3, \ldots, n$ is a ________ estimator of population mean.

(5) If T_n is an estimator of a parameter θ of the density $f(x;\theta)$ the quantity $E\left[\frac{\partial}{\partial\theta}\log f(x;\theta)\right]^2$ is called the ________.

(6) If $S = s(X_1, X_2, X_3, \ldots, X_n)$ is a sufficient statistic for θ of density $f(x;\theta)$ and $f(x_i;\theta)$ for $i = 1, 2, 3, \ldots, n$ can be factorized as $g(s,\theta)h(x)$, then $s(X_1, X_2, X_3, \ldots, X_n)$ is a ________

(7) If $f(x;\theta)$ is a family of distributions and $h(x)$ is any statistic such that $E[h(x)] = 0$, then $f(x;\theta)$ is called ________

(8) If a random sample $x_1, x_2, x_3, ..., x_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum liokelihood estimate of μ is ________

(9) For a rectangular distribution $\frac{1}{(\beta - \alpha)}$, the maximum likelihood estimates of a and β are ________ and ________ respectively.

(10) Let $x_1, x_2, x_3, ..., x_n$ be a random sample from a density $f(x, \theta) = \theta e^{-\theta x}$. Then the Crammer-Raolower bound of variance of unbiased estimator is ________

(11) Relative efficiency of an estimator T_n as compared to an estimator is T_n' given as ________

(12) The estimate of the parameter λ of the exponential distribution $\lambda e^{-\lambda x}$ by the method of moments is ________

(13) Maximum likelihood estimate of the parameter θ of the distribution $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}$ is ________.

(14) If $x_1, x_2, x_3, ..., x_n$ is a random sample from an infinite population and s^2 is defined as $\frac{\sum (x_i - \bar{x})^2}{n}, \frac{n}{n=1} s^2$ is an ________ estimator of population variance σ^2.

(15) $\frac{\bar{x}}{k} + 1$ is an unbiased estimator of $\frac{1}{p}$ in Negative Binomial distribution.

(16) Write the statement of Factorization theorm.

(17) Name different criteria of good estimators.

(18) Write likelihood function of $f(x, \theta) = \theta e^{-x\theta}; 0 \leq x \leq \infty.$

(19) Write likelihood function of $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}$

(20) Obtain Cramer-Rao lower bound of variance of unbiased estimator of parameter of $f(x, \theta) = \frac{\theta^x}{x!} e^{-\theta}; x = 0, 1, ..., \infty; \theta > 0.$

2 (A) Write the answer any **Three** : **6**

(1) Define Parameter space.
(2) Define Efficiency.
(3) Define Sufficiency
(4) Define Most Powerful Test (MP test)
(5) Define Average Sample Number function of SPRT
(6) Obtain likelihood function of Binomial distribution.

(B) Write the answer any **Three** : **9**

(1) Show that $\frac{x(x-1)}{n(n-1)}$ is a unbiased estimator p^2 of Binomial distribution.

(2) If $x_1, x_2, x_3, \ldots x_n$ random sample taken from distribution with mean θ and variance σ^2 then $t_1 = \frac{\sum x_i}{n+1}$ is a consistent of θ. Check it?

(3) Obtain MVUE of parameter θ for Poisson distribution.

(4) Obtain estimator of θ by method of moments in the following distribution $f(x;\theta) = \theta x^{\theta-1}; 0 \le x \le 1$

(5) Give a random sample $x_1, x_2, x_3, \ldots, x_n$ from distribution with p.d.f. $f(x;\theta) = \frac{1}{\theta}; 0 \le x \le \theta.$ Obtain power of the test for testing $H_o : \theta = 1.5$ against $H_1 : \theta = 2.5$ where $c = \{x; x \ge 0.8\}$.

(6) Obtain Operating Characteristic (OC) function of SPRT.

(C) Write the answer any **Two** : **10**

(1) State Crammer-Rao inequality and prove it.

(2) Estimate α and β in the case of Gamma distribution by the method of moments

$$f(x;\alpha,\beta) = \frac{\alpha^\beta}{\Gamma\beta} e^{-ax} x^{\beta-1}; x \ge 0, a \ge 0$$

(3) Obtain OC function for SPRT of Binomial distribution for testing $H_0 : p = p_0$ against $H_1 : p = p_1 (> p_0)$

(4) Give a random sample $x_1, x_2, x_3, \ldots x_n$ from distribution with p.d.f. $f(x;\theta) = \theta e^{-\theta x}; 0 \le x \le \infty, \theta > 0$ Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(5) If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2, σ_2^2 and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination?

3 (A) Write the answer any **Three** : **6**

(1) Define Consistency

(2) Define Complete family of distribution

(3) Define Minimum Variance Bound Estimator (MVBE)

(4) Define Uniformly Most Powerful Test (UMP test)

(5) Obtain an unbiased estimator of θ by for the following distribution $f(x;\theta) = \theta e^{-x\theta}; 0 \le x \le \infty$

(6) Show that sample mean is more efficient than sample median for Normal distribution.

(B) Write the answer any Three : **9**

(1) Let $x_1, x_2, x_3, ..., x_n$ be random sample taken from $N(\mu, \sigma^2)$ then find sufficient estimator of μ and σ^2.

(2) Obtain an unbiased estimator of population mean of χ^2 distribution.

(3) Prove that $E\left(\frac{\partial \log L}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)$

(4) If A is more efficiency than B then prove that $Var(A) + Var(B - A) = Var(B)$

(5) Explain method of minimum χ^2.

(6) Let p be the probability that coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{2}$. the coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type-I, type-II and power of test.

(C) Write the answer any **Two** : **10**

(1) State Neyman-Pearson lemma and prove it.

(2) Obtain MVBE of σ^2 for Normal distribution.

(3) Obtain Likelihood Ration Test

(4) For the double Poisson distribution

$$P(X = x) = \frac{1}{2}\frac{e^{-m_1} m_1^{\,x}}{x!} + \frac{1}{2}\frac{e^{-m_2} m_2^{\,x}}{x!}; 0, 1, 2, ...$$ Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - (\mu_1')^2}$

(5) Construct SPRT of Binomial distribution for testing $H_0 : p = p_0$ against $H_1 : p = p_1 (> p_0)$. Also obtain OC function of SPRT.